



24
DIAGRAMS

SIGMA PREP

The MUST-know SAT Math formulas & concepts

Everything the test expects you to know and does not print on its reference sheet. One idea per page, each one solved on a real SAT question, with a code that opens the video.

How to use this

- 1 Learn the diagrams by using them through practice problems.
- 2 Use these examples to understand the concepts. Scan the QR code to watch a more in-depth explanation that uses Desmos.
- 3 Try similar problems in the app to prove you can do it on your own.

Every question here comes from the Sigma Prep practice banks, where each skill has easy, medium, and hard sets with a video explanation on every problem. The free full-length test sorts your misses into these pages for you.

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6 diagrams

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The SAT Math section tests 19 skills in four areas. These 24 diagrams cover 16 of them. Under each title is the College Board skill name, so a miss on a practice test points you to a page. Every diagram is followed by a real question solved with it; scan the code on that page to watch it solved.

Slope Intercept Form Interpretation

$$y = \underline{mx} + \underline{b}$$

slope

rate of change

increase/decrease

keywords "per"
and "each"

y-intercept

initial value

starting point

y-value when $x=0$



NOTES

Slope is always attached to the variable, and the keywords are per and each. Slope is the change in y for ONE change in x .

Example on the next page →

Try it: Slope-Intercept Form Interpretation

A hard question from the Linear functions bank. Solve it with the diagram on the facing page, then check the steps.

A container is being filled with water. The formula $h = 40 + 5t$ relates the height h , in centimeters, of the water in the container to the time t , in minutes, since filling began. Which of the following describes the meaning of the 5 in this context?

- A** The time, in minutes, to reach a height of 40 centimeters
- B** The increase in the height, in centimeters, of the water for each one-minute increase in time
- C** The height, in centimeters, of the water when filling began
- D** The increase in time, in minutes, for each one-centimeter increase in the height of the water

HOW THE DIAGRAM APPLIES

1. Match it to $y = mx + b$: the slope m is 5, the intercept b is 40.
2. The slope is the rate of change: how much h changes for each one-unit increase in t . So 5 is centimeters per minute.
3. 40 is the starting point, the height when filling began. That is choice C, the other number.

ANSWER **B** The increase in the height, in centimeters, of the water for each one-minute increase in time



Watch it solved

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Parallel and Perpendicular

$$y = \frac{3}{5}x + 4$$

slope

parallel
- same slope

$$\text{slope} = 3/5$$

perpendicular
- opposite reciprocal slope

$$\text{slope} = -5/3$$



NOTES

You can only read the slope in slope-intercept form, so isolate y first.
Perpendicular is opposite reciprocal: flip the sign *AND* flip the fraction.

Example on the next page →

Try it: Parallel and Perpendicular

A hard question from the Linear equations in two variables bank. Solve it with the diagram on the facing page, then check the steps.

What is the slope of a line that is perpendicular to the line defined by the equation $2y + 14x = 9$ in the xy -plane?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. Get the line into $y = mx + b$: $2y = -14x + 9$, so $y = -7x + \frac{9}{2}$. The slope is -7 .
2. Perpendicular means opposite reciprocal: flip -7 to $-\frac{1}{7}$ and change the sign.
3. The slope is $\frac{1}{7}$.

ANSWER **$\frac{1}{7}$**

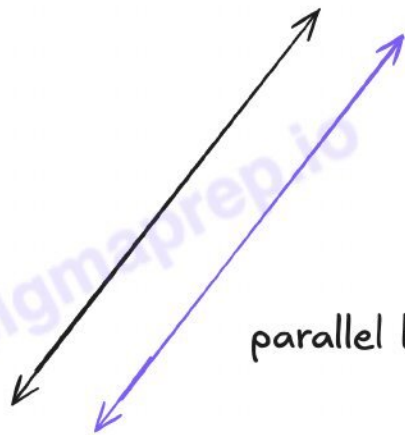


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No Solution and Infinite Solutions

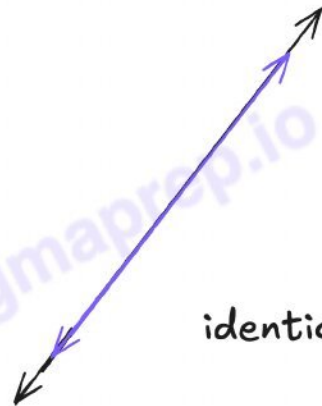


parallel lines

As Equations

$$3x + 4 = 3x + 2$$

same slope
different y-intercept



identical lines

As Equations

$$5x + 1 = 5x + 1$$

same slope
same y-intercept → identical

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NOTES

No solution means parallel lines: same slope, different y-intercept. Infinitely many solutions means the same line: same slope and the same y-intercept. Get both sides into slope-intercept form, then match the slopes (and, for infinite, the intercepts) and solve for the constant.

Example on the next page →

Try it: No Solution and Infinite Solutions

A hard question from the Linear equations in one variable bank. Solve it with the diagram on the facing page, then check the steps.

In the given equation, k is a constant. The equation has no solution. What is the value of k ?

$$4(kx + 7) = \frac{52}{11}x + 30$$

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. Distribute: $4kx + 28 = \frac{52}{11}x + 30$.
2. No solution means the two sides are parallel lines: same slope, different intercept. The intercepts 28 and 30 already differ, so set the slopes equal.
3. $4k = \frac{52}{11}$, so $k = \frac{13}{11}$.

ANSWER **13/11**



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Mixture Problem

$$\begin{array}{c} \% \\ * \\ \text{Amount} \end{array} \boxed{} + \boxed{} = \boxed{}$$



NOTES

Two cups going into one cup. Every cup gets a percent and an amount, and the equation is $(\text{percent})(\text{amount}) + (\text{percent})(\text{amount}) = (\text{percent})(\text{amount})$. If one amount is x , the other one is the total minus x .

Example on the next page →

Try it: Mixture Problem

A hard question from the Systems of two linear equations bank. Solve it with the diagram on the facing page, then check the steps.

A sample of a certain solution has a total mass of 60.0 grams and is 40.0% salt by mass. The sample was created by combining two portions of different solutions. The first portion was 25.0% salt by mass and the second portion was 70.0% salt by mass. What was the mass, in grams, of the second portion?

A 20.0

B 14.0

C 40.0

D 28.0

HOW THE DIAGRAM APPLIES

1. Fill the boxes: percent times amount for each portion, equal to percent times the total. Let x be the second portion, so the first is $60 - x$.
2. $0.25(60 - x) + 0.70x = 0.40(60)$
3. $15 + 0.45x = 24$, so $x = 20$ grams. Keep it as x and Desmos solves it.

ANSWER **A** 20.0



Watch it solved

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Exponent Rules

1) Multiplying terms

$$x^2 * x^5 = x^{2+5} = x^7$$

2) Dividing terms

$$\frac{x^9}{x^4} = x^{9-4} = x^5$$

3) Power to Power

$$(x^3)^6 = x^{3*6} = x^{18}$$



NOTES

Three rules, in order. Multiplying means add the exponents, dividing means subtract the exponents, and an exponent raised to another exponent means you multiply the exponents.

Example on the next page →

Try it: Exponent Rules

A medium question from the Equivalent expressions bank. Solve it with the diagram on the facing page, then check the steps.

Which expression is equivalent to $\frac{h^{12}q^5}{h^4q^{17}}$, where $h > 0$ and $q > 0$?

A $\frac{h^8}{q^{12}}$

B h^8q^{12}

C $\frac{q^{12}}{h^8}$

D $h^{16}q^{22}$

HOW THE DIAGRAM APPLIES

1. Dividing like bases: subtract the exponents, one base at a time.
2. $h^{12-4} = h^8$ and $q^{5-17} = q^{-12}$.
3. A negative exponent moves the term to the bottom: $\frac{h^8}{q^{12}}$.

ANSWER **A** $\frac{h^8}{q^{12}}$



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Exponential to Radical

Form

$$x^{\frac{\text{power}}{\text{root}}}$$



Form

$$\sqrt[\text{root}]{x^{\text{power}}}$$



NOTES

Never leave a radical as a radical. Root on the outside, power on the inside, and as an exponent you write power divided by root. Also remember that a square root has a root of 2 even though nobody writes a 2 on the outside.

Example on the next page →

Try it: Exponential to Radical

A hard question from the Equivalent expressions bank. Solve it with the diagram on the facing page, then check the steps.

If $\frac{\sqrt{x^7}}{\sqrt[3]{x^5}} = x^{\frac{a}{b}}$ for all positive values of x , where $\frac{a}{b}$ is in lowest terms, what is the value of $\frac{a}{b}$?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. Radical to exponent, power over root: $\sqrt{x^7} = x^{7/2}$ and $\sqrt[3]{x^5} = x^{5/3}$.
2. Dividing like bases: subtract the exponents. $\frac{7}{2} - \frac{5}{3} = \frac{21}{6} - \frac{10}{6} = \frac{11}{6}$.
3. $\frac{a}{b} = \frac{11}{6}$.

ANSWER **11/6**



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Factoring

$$\begin{array}{ccc} \underline{x^2} & + & 2x & - & \underline{15} \\ \text{First} & & & & \text{Last} \end{array}$$

$$\begin{array}{cccc} \underline{(x-3)} & \underline{(x+5)} \\ \text{First} & & \text{Last} \end{array}$$

 SigmaPrep**NOTES**

When comparing factored form to expanded form, notice that in FOIL first times first gives the first term in expanded form, and last times last gives the last term.

Example on the next page →

Try it: Factoring

A medium question from the Equivalent expressions bank. Solve it with the diagram on the facing page, then check the steps.

If the expression $3x^2 + 7x - 6$ is rewritten in the form $(3x - 2)(x + k)$, where k is a constant, what is the value of k ?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. The two numbers in the factors multiply to the LAST term: $(-2)(k) = -6$, so $k = 3$.
2. Check the middle with First and Last: $3x \cdot 3 + (-2) \cdot x = 9x - 2x = 7x$. It matches.

ANSWER **3**

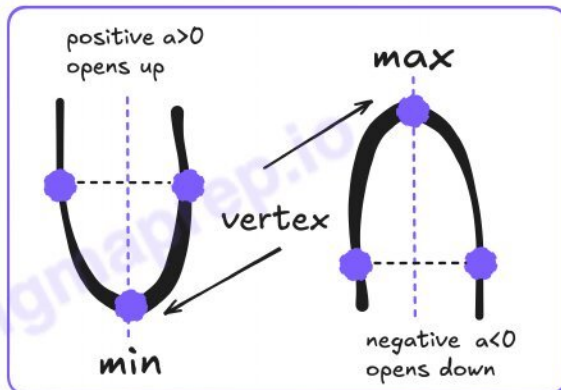


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Quadratics



Common Questions

Initial height = y-intercept

Hits the ground = x-intercepts

Maximum/minimum = vertex

3 Quadratic Forms

1) Standard Form

$$y = ax^2 + bx + c$$

know the y-int $y=c$

2) Vertex Form

$$y = a(x-h)^2 + k$$

know the vertex (h,k)

3) Factored Form

$$y = a(x-3)(x+5)$$

know x-intercepts $x=3$, $x=-5$

NOTES

Know the 3 quadratic forms and what information each one gives you.

Know the common questions and what they represent in context.

Understand symmetry through the vertex of a quadratic.

Example on the next page →

Try it: Quadratics

A hard question from the Nonlinear functions bank. Solve it with the diagram on the facing page, then check the steps.

Function f is a quadratic function where $f(-18) = 0$ and $f(-6) = 0$. The graph of $y = f(x)$ in the xy -plane has a vertex at $(r, -36)$. What is the value of r ?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. $f(-18) = 0$ and $f(-6) = 0$ are the x -intercepts, so factored form is $f(x) = a(x + 18)(x + 6)$.
2. A parabola is symmetric: the vertex sits exactly halfway between the x -intercepts.
3. $r = \frac{-18 + (-6)}{2} = -12$.

ANSWER **-12**

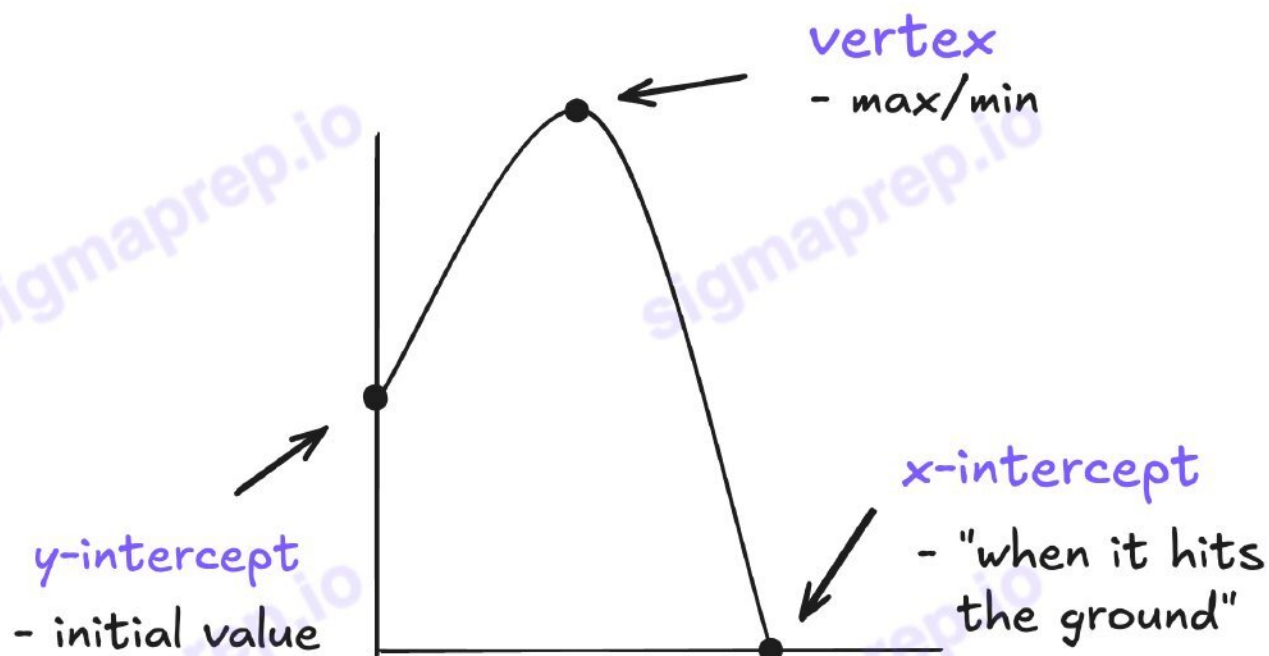


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Quadratic Interpretations



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NOTES

Three points they ask about: the y-intercept is the "starting value," the x-intercept is when it "hits the ground," the vertex is the "max or min." Those are the kinds of words to listen for; they are not always word for word.

Example on the next page →

Try it: Quadratic Interpretations

A hard question from the Nonlinear functions bank. Solve it with the diagram on the facing page, then check the steps.

The given function $f(x)$ gives a metal ball's height above the ground $f(x)$, in inches, x seconds after it started moving on a track, where $0 \leq x \leq 20$. Which of the following is the best interpretation of the vertex of the graph of $y = f(x)$ in the xy -plane?

$$f(x) = \frac{1}{16}(x - 5)^2 + 4$$

- | | |
|--|---|
| A The metal ball's height was 5 inches when it started moving | B The metal ball's minimum height was 5 inches |
| C The metal ball's height was 4 inches when it started moving | D The metal ball's minimum height was 4 inches |

HOW THE DIAGRAM APPLIES

1. $f(x) = \frac{1}{16}(x - 5)^2 + 4$ is vertex form, so the vertex is $(5, 4)$: know the vertex (h, k) .
2. $a = \frac{1}{16} > 0$, so the parabola opens up and the vertex is a minimum.
3. The minimum height was 4 inches, and it happened at $x = 5$ seconds. The question asks for the height, the y of the vertex.

ANSWER **D** The metal ball's minimum height was 4 inches



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Discriminant

$$b^2 - 4ac$$

discriminant is
part of the
quadratic formula

a, b, c come from
 $ax^2 + bx + c = 0$

3 cases to know

i) $b^2 - 4ac > 0$
2 solutions

ii) $b^2 - 4ac = 0$
1 solution

iii) $b^2 - 4ac < 0$
No real solution

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NOTES

Two things to spot: you need a quadratic, and a sentence telling you how many solutions. That is a discriminant problem. Know the 3 cases: greater than 0 for 2 solutions, equal to 0 for 1 solution, and less than 0 for no solution. Then let Desmos solve it.

Example on the next page →

Try it: Discriminant

A hard question from the Nonlinear equations in one variable and systems of equations in two variables bank. Solve it with the diagram on the facing page, then check the steps.

In the given equation, c is a constant. The equation has exactly one solution. What is the value of c ?

$$-3x^2 + 24x + c = 0$$

A -96

B -48

C -32

D 0

HOW THE DIAGRAM APPLIES

1. Read off $a = -3$, $b = 24$, $c = c$. Everything is already on one side.
2. Exactly one solution is case ii: $b^2 - 4ac = 0$.
3. $576 - 4(-3)(c) = 576 + 12c = 0$, so $c = -48$.

ANSWER **B** -48



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Exponential Equation

$$y = \underline{a}(\underline{1 \pm r})^x$$

↗
y-intercept
initial value

↖ $r\%$
increase $\rightarrow 1 + r$
decrease $\rightarrow 1 - r$

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NOTES

Two things to know: the number in front is the initial value, and the number inside the parentheses, with the exponent, represents the increase or decrease (usually in terms of percent).

Example on the next page $\rightarrow$

Try it: Exponential Equation

A hard question from the Nonlinear functions bank. Solve it with the diagram on the facing page, then check the steps.

For the function f , $f(0) = 75$, and for each increase in x by 1, the value of $f(x)$ decreases by 60%. What is the value of $f(2)$?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. a is the value at $x = 0$: $a = 75$. Decrease of 60% means multiply by $1 - 0.60 = 0.40$ each step.
2. $f(x) = 75(0.4)^x$
3. $f(2) = 75(0.4)^2 = 75(0.16) = 12$.

ANSWER **12**



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Transformations

translations / shifts

Vertical

think outside

$$y = f(x) + 2$$

shift up 2 units

$$y = f(x) - 3$$

shift down 3 units

Horizontal

(think inside)

$$y = f(x + 4)$$

shift left 4 units

$$y = f(x - 5)$$

shift right 5 units



NOTES

They only really test you on shifts. Vertical, think outside. Horizontal, think inside. Vertical is intuitive: $f(x) + 5$ means up 5. Horizontal is counterintuitive: $f(x + 5)$ means left 5, not right 5.

Example on the next page →

Try it: Transformations

A hard question from the Nonlinear functions bank. Solve it with the diagram on the facing page, then check the steps.

The function f is defined by the given equation. The function g is defined by $g(x) = f(x + 3)$. For what value of x does $g(x)$ reach its minimum?

$$f(x) = 3x^2 + 42x + 150$$

A -7

B -3

C -10

D -4

HOW THE DIAGRAM APPLIES

1. Find where f has its minimum first: $x = -\frac{b}{2a} = -\frac{42}{6} = -7$.
2. $g(x) = f(x + 3)$: inside the parentheses, so it moves the OPPOSITE way. Plus 3 shifts the graph LEFT 3.
3. The minimum moves from $x = -7$ to $x = -10$.

ANSWER C -10



Watch it solved

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Unit Conversions

Convert 5 feet per minute into yards per hour

(3 feet = 1 yard)

Conversion Factor

$$\frac{5 \cancel{\text{ feet}}}{1 \cancel{\text{ minute}}} \times \frac{1 \cancel{\text{ yard}}}{3 \cancel{\text{ feet}}} \times \frac{60 \cancel{\text{ minutes}}}{1 \text{ hour}} = \frac{300 \text{ yards}}{3 \text{ hours}}$$

$$\searrow = \frac{100 \text{ yards}}{1 \text{ hour}}$$



NOTES

Per makes the fraction. Line the units up so each one cancels: whatever is on the bottom of one fraction goes on top of the next. If a unit is squared, square the whole conversion factor, or the units will not cancel.

Example on the next page →

Try it: Unit Conversions

A hard question from the Ratios, rates, proportional relationships, and units bank. Solve it with the diagram on the facing page, then check the steps.

The speed of a sled is increasing at a rate of 8.6 meters per second squared. What is this rate, in miles per minute squared, rounded to the nearest tenth? (Use 1 mile = 1,609 meters.)

A 230.6

B 19.2

C 0.3

D 3.8

HOW THE DIAGRAM APPLIES

1. Chain the fractions so each unwanted unit cancels: $\frac{8.6 \text{ m}}{\text{s}^2} \times \frac{1 \text{ mi}}{1,609 \text{ m}} \times \left(\frac{60 \text{ s}}{1 \text{ min}}\right)^2$
2. Seconds are squared, so the seconds-to-minutes factor is applied twice: $60^2 = 3,600$.
3. $\frac{8.6 \times 3,600}{1,609} \approx 19.2$ miles per minute squared.

ANSWER **B** 19.2



Watch it solved

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Percent Change

$$\text{final} = \text{original} * (1 \pm r)$$

↑
final
value

↑
original
value

↑
r %
1 + r → increase
1 - r → decrease

Can use this to find 3 things:

- 1) the final value
- 2) the original value
- 3) the percent (r)



NOTES

Use this equation in 3 different ways: to find the final amount, the original amount, or the percent. Make whatever the question asks for the unknown, x , plug in numbers for the other two, and let Desmos solve it for you. Very useful and versatile.

Example on the next page →

Try it: Percent Change

A medium question from the Percentages bank. Solve it with the diagram on the facing page, then check the steps.

A number n is increased by 5%. If the result is 378, what is the value of n ?

A 360

B 252

C 359

D 397

HOW THE DIAGRAM APPLIES

1. This is the "find the original value" case: $\text{final} = \text{original} \times (1 + r)$.
2. $378 = n(1.05)$, so $n = \frac{378}{1.05} = 360$.
3. The trap is choice C: taking 5% off 378 gives 359.1, which is not the original.

ANSWER **A** 360



Watch it solved

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Statistics

5 things to know

1) mean = average = $\frac{\text{sum}}{\text{\# of numbers}}$

4) mode = most common number

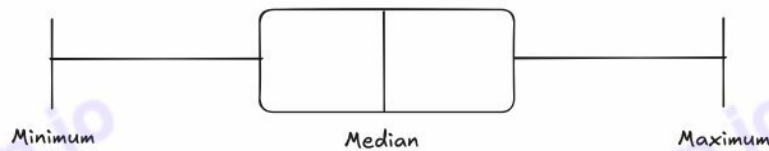
2) median = middle number of an ordered list

5) standard deviation = measures how far values typically vary from the mean (think "spread")

3) range = largest - smallest

Note: mean, median, standard deviation can be done in desmos

Box Plot



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NOTES

Know these 5 calculations for statistics. Mean, median, and standard deviation can be computed in Desmos. Range and mode **MUST** be memorized, because Desmos will not do them for you.

Example on the next page →

Try it: Statistics

A medium question from the new Problem-Solving and Data Analysis set. Solve it with the diagram on the facing page, then check the steps.

$d, 14, 18, e, 27, 39, f$

For the given data set, the data values are listed in ascending order, where d , e , and f are constants. For this data set, the mean is 24, the median is 21, and the range is 51. What is the value of f ?

- | | |
|-------------|-------------|
| A 49 | B 50 |
| C 51 | D 99 |

HOW THE DIAGRAM APPLIES

1. Seven values in order, so the median is the 4th value: $e = 21$.
2. Mean 24 means the sum is $7 \times 24 = 168$. The five known values add to $14 + 18 + 21 + 27 + 39 = 119$, so $d + f = 49$.
3. Range 51 means $f - d = 51$. Add the two equations: $2f = 100$, so $f = 50$.

ANSWER **B** 50



Watch it solved

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Standard Deviation

- measures how far values typically vary from the mean
- think "spread"
- more "spread" = larger standard deviation

Example 1:

A: 1, 2, 3, 4, 5

B: 10, 20, 30, 40, 50

Example 2:

C: 1, 2, 3, 4, 5

D: 101, 102, 103, 104, 105



NOTES

For standard deviation, simply think of spread. Just remember that one word. In the examples above, B has more spread than A, so B has the larger standard deviation. C and D always go up by 1, so their spreads are the same.

Example on the next page →

Try it: Standard Deviation

A medium question from the new Problem-Solving and Data Analysis set. Solve it with the diagram on the facing page, then check the steps.

Which of the following lists represents a data set with the largest standard deviation?

A 70, 70, 70, 70, 70

B 68, 69, 70, 71, 72

C 62, 66, 70, 74, 78

D 40, 55, 70, 85, 100

HOW THE DIAGRAM APPLIES

1. Standard deviation is spread from the mean, and every list here has mean 70.
2. List A has no spread at all. B steps by 1, C by 4, D by 15.
3. The most spread out list has the largest standard deviation: D.

ANSWER **D** 40, 55, 70, 85, 100



Watch it solved

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Probability

$$\text{probability} = \frac{\text{favorable}}{\text{total}}$$

total

if a cat is selected at random what is the probability it is large?

favorable

	small	medium	large	total
cat	17	12	6	35
dog	23	17	20	60
total	40	29	26	95

What is the probability that the animal is large given that it is a cat?

favorable

total



NOTES

Probability just wants you to create a fraction. When you read these problems you need to know what the denominator is (the total) and what the numerator is (the favorable). The phrase "given that" swaps the total around.

Example on the next page →

Try it: Probability

A medium question from the new Problem-Solving and Data Analysis set. Solve it with the diagram on the facing page, then check the steps.

For 100 bolts, the table summarizes the distribution by machine and length.

Machine	Length (millimeters)		
	Less than 15	15 to 25	Greater than 25
Machine 1	14	9	2
Machine 2	0	12	21
Machine 3	13	29	0

One of these bolts will be selected at random. What is the probability of selecting a bolt with a length that is less than or equal to 25 millimeters, given that it was **not** made by Machine 2? (Express your answer as a decimal or fraction, not as a percent.)

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. "Given that it was not made by Machine 2" shrinks the total to the Machine 1 and Machine 3 rows: $25 + 42 = 67$ bolts. The grand total of 100 is gone.
2. Favorable, inside those two rows only: length ≤ 25 is $14 + 9 + 13 + 29 = 65$.
3. $\frac{65}{67}$

ANSWER **65/67**



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Statistical Inference

margin of error is the possible error
(think $\pm$)

example: 53% of voters voted for
plan A with a margin of error of 2%

↳ means $53\% \pm 2\%$ voted for plan A

↳ likely to be 51% to 55%
that voted for plan A

Extrapolation

example: Out of 40 people surveyed 21
people voted for plan A. If there are
2,000 residents how many are expected
to vote for plan A?

↳ Solve the proportion $\frac{21}{40} = \frac{x}{2000}$
 $x = 1,050$



NOTES

Margin of error is a plus or minus around the reported number: the true value is most likely between the estimate minus the margin and the estimate plus the margin. If they give you the range instead, the margin is the distance from the middle to one end, not the whole width. For sample-to-population questions, set up the proportion and scale it up to the population.

Example on the next page →

Try it: Statistical Inference

A medium question from the new Problem-Solving and Data Analysis set. Solve it with the diagram on the facing page, then check the steps.

Researchers studied a sample of a species of moth from a certain forest. The researchers found that the mean wingspan of moths in the sample was 24.83 millimeters. Based on the sample mean and margin of error, the researchers estimated that the mean wingspan of all moths of this species from this forest was between 23.96 and 25.70 millimeters. What is the margin of error, in millimeters?

- A** 0.87 **B** 1.74
C 23.96 **D** 24.83

HOW THE DIAGRAM APPLIES

1. The estimate sits in the middle of the interval: 24.83. The margin of error is the $\pm$, the distance from the middle to either end.
2. $24.83 - 23.96 = 0.87$, and $25.70 - 24.83 = 0.87$.
3. Choice B, 1.74, is the whole width of the interval, not the $\pm$.

ANSWER **A** 0.87



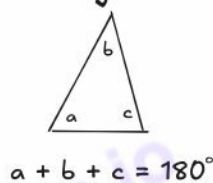
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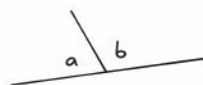
Angle Relationships

Triangle Sum



$$a + b + c = 180^\circ$$

Linear Pair



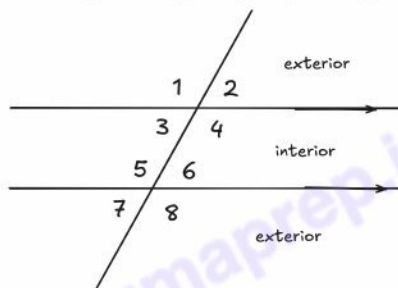
$$a + b = 180^\circ$$

Vertical Angles



$$a = b$$

Parallel Line Relationships



- 1) Alternate Interior Angles
 $3 = 6$ and $4 = 5$
- 2) Alternate Exterior Angles
 $1 = 8$ and $2 = 7$
- 3) Corresponding Angles
 $1 = 5$, $3 = 7$,
 $2 = 6$, and $4 = 8$
- 4) Same Side Interior Angles
 $3 + 5 = 180^\circ$
 $4 + 6 = 180^\circ$

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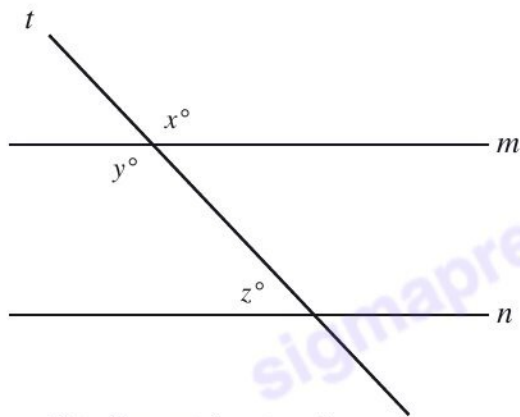
NOTES

When they say parallel, you are using one of the parallel-line relationships: alternate interior, alternate exterior, corresponding, or same-side interior angles. The other relationships shown here do not require parallel lines and are always true: vertical angles, linear pair, and triangle sum.

Example on the next page →

Try it: Angle Relationships

A medium question from the Lines, angles, and triangles bank. Solve it with the diagram on the facing page, then check the steps.



Note: Figure not drawn to scale.

In the figure, lines m and n are parallel. If $x = 10k - 11$ and $y = 8k + 15$, what is the value of z ?

- | | |
|--------------|-------------|
| A 180 | B 61 |
| C 119 | D 13 |

HOW THE DIAGRAM APPLIES

1. x and y are vertical angles, so they are equal: $10k - 11 = 8k + 15$, giving $k = 13$ and $x = y = 119$.
2. y and z are same-side interior angles, so they ADD to 180. They are not equal.
3. $z = 180 - 119 = 61$.

ANSWER **B** 61



Watch it solved

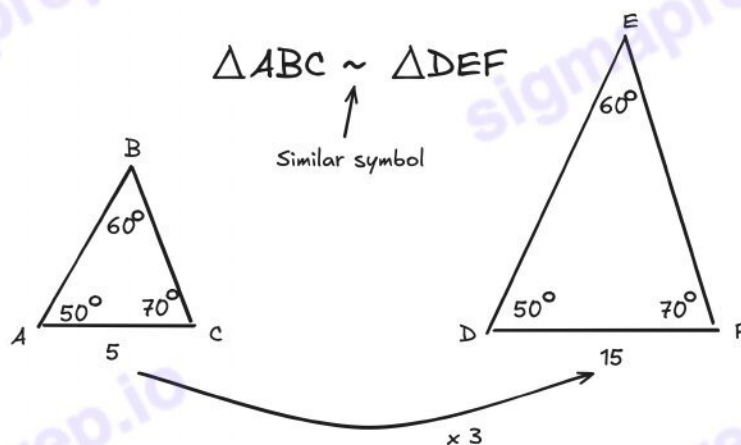
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Similar Figures

3 Properties

- 1) Corresponding angles are equal
- 2) Corresponding sides are proportional
- 3) Corresponding angles have the same trig ratios



Compare two corresponding sides to determine the proportionality



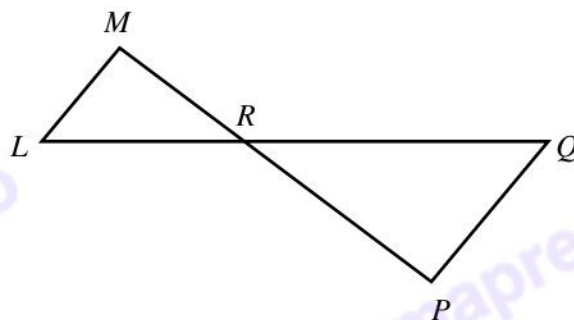
NOTES

Similar figures are very common on the test, and it is super important that you always remember the first two properties. The third property, the one about trig, is not always used, but it is nice to know.

Example on the next page →

Try it: Similar Figures

A hard question from the Lines, angles, and triangles bank. Solve it with the diagram on the facing page, then check the steps.



Note: Figure not drawn to scale.

In the figure, $\overline{LQ}$ intersects $\overline{MP}$ at point R , and $\overline{LM}$ is parallel to $\overline{PQ}$. The lengths of $\overline{MR}$, $\overline{LR}$, and $\overline{RP}$ are 7, 9, and 12, respectively. What is the length of $\overline{LQ}$?

- | | |
|--------------------------|--------------------------|
| A 21 | B $\frac{108}{7}$ |
| C $\frac{171}{7}$ | D $\frac{55}{3}$ |

HOW THE DIAGRAM APPLIES

1. $LM \parallel PQ$, so the alternate interior angles are equal ($\angle L = \angle Q$, $\angle M = \angle P$) and the vertical angles at R are equal. Triangles LMR and QPR are similar.
2. Match sides by the angles they sit across from: MR corresponds to PR , so the scale factor is $\frac{12}{7}$. Then LR corresponds to QR : $QR = 9 \cdot \frac{12}{7} = \frac{108}{7}$.
3. $LQ = LR + RQ = 9 + \frac{108}{7} = \frac{171}{7}$.

ANSWER **C** $\frac{171}{7}$

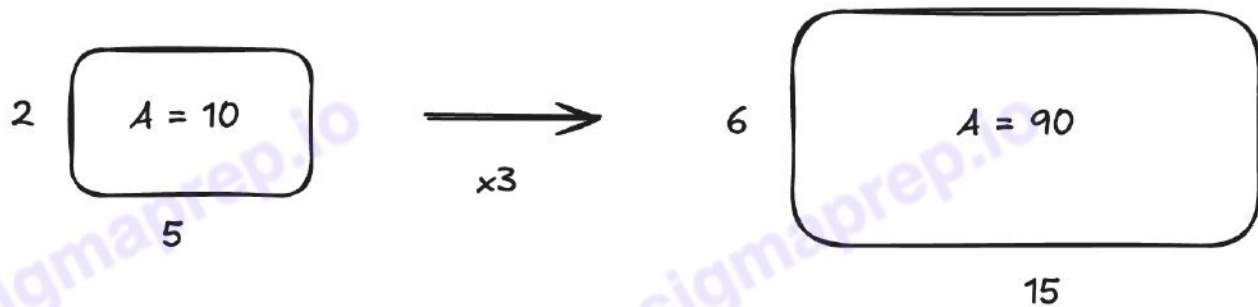


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Proportionality



sides are 3 times bigger

area is 9 times bigger or $(3)^2$ bigger

volume would be 27 times bigger or $(3)^3$ bigger



NOTES

If the sides are k times bigger, the area is k squared times bigger and the volume is k cubed times bigger. Area is units squared, volume is units cubed. One calculation, not a whole recomputation.

Example on the next page →

Try it: Proportionality

A hard question from the Area and volume bank. Solve it with the diagram on the facing page, then check the steps.

The length of each side of square A is 134 times the length of each side of square B . The area of square A is k times the area of square B . What is the value of k ?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. Sides scale by k , area scales by k^2 .
2. Sides are 134 times bigger, so the area is $134^2 = 17,956$ times bigger.

ANSWER **17956**



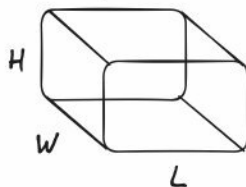
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Volume and Surface Area

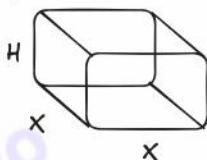
Rectangular Prism



$$\text{Volume} = LWH$$

$$\text{Surface Area} = 2LW + 2WH + 2LH$$

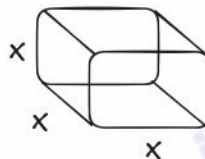
Square Prism



$$\text{Volume} = X^2 H$$

$$\text{Surface Area} = 2X^2 + 4XH$$

Cube



$$\text{Volume} = X^3$$

$$\text{Surface Area} = 6X^2$$

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NOTES

The SAT gives you the rectangular prism on its reference sheet, but it does not give you surface area. You need to be able to derive surface area by looking at the figure, or memorize it from here. The square prism and the cube are variations of a rectangular prism, with two sides the same and all three sides the same, respectively.

Example on the next page →

Try it: Volume and Surface Area

A hard question from the Area and volume bank. Solve it with the diagram on the facing page, then check the steps.

A cube has a volume of 262,144 cubic units. What is the surface area, in square units, of this cube?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. Cube: Volume = x^3 , so $x^3 = 262,144$ and $x = 64$ (check: $64 \cdot 64 \cdot 64 = 262,144$).
2. Surface area = $6x^2 = 6(64)^2 = 6(4,096) = 24,576$.

ANSWER **24576**



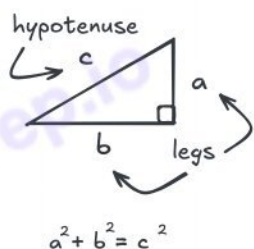
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Trigonometry

Pythagorean Theorem



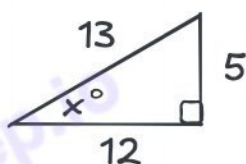
SOH CAH TOA

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}}$$

example



$$\sin x^\circ = \frac{5}{13} \quad \tan x^\circ = \frac{5}{12}$$

$$\cos x^\circ = \frac{12}{13}$$

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NOTES

SOH CAH TOA and the Pythagorean theorem **ONLY** apply to right triangles. Use the Pythagorean theorem to find a missing side. Trig questions just want a ratio.

Example on the next page →

Try it: Trigonometry

A hard question from the Right triangles and trigonometry bank. Solve it with the diagram on the facing page, then check the steps.

In triangle PQR , $\cos(Q) = \frac{14}{50}$ and angle P is a right angle. What is the value of $\cos(R)$?

STUDENT-PRODUCED RESPONSE · TYPE THE NUMBER

HOW THE DIAGRAM APPLIES

1. P is the right angle, so Q and R are the two acute angles and $Q + R = 90^\circ$. That makes $\cos(R) = \sin(Q)$.
2. $\cos(Q) = \frac{14}{50}$ means adjacent 14, hypotenuse 50. Pythagorean theorem: opposite = $\sqrt{50^2 - 14^2} = \sqrt{2,304} = 48$.
3. $\cos(R) = \sin(Q) = \frac{48}{50} = 0.96$.

ANSWER **0.96**



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Circles

Circle Equation

$$(x - h)^2 + (y - k)^2 = r^2$$

center = (h, k)

radius = r

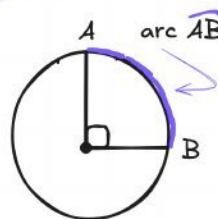
Unit Conversion

$$180^\circ = \pi$$

(degrees) (radians)

Arc Length

a piece of the circumference



arc AB is $\frac{1}{4}$ of the circumference

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NOTES

The right side of $(x - h)^2 + (y - k)^2 = r^2$ is the radius SQUARED. A radius of 3m makes the right side $9m^2$, not $3m^2$, and a right side of $49k^2$ means the radius is 7k. The center goes in with the signs flipped: center $(5, 12)$ is $(x - 5)$ and $(y - 12)$. Two traps in one question, and both wrong answers are sitting in the choices.

Example on the next page →

Try it: Circles

A hard question from the Circles bank. Solve it with the diagram on the facing page, then check the steps.

A circle in the xy -plane has its center at $(5, 12)$ and has a radius of $3m$. Which of the following is the equation of this circle?

A $(x + 5)^2 + (y + 12)^2 = 9m^2$

B $(x - 5)^2 + (y - 12)^2 = 9m^2$

C $(x - 5)^2 + (y - 12)^2 = 3m^2$

D $(x + 5)^2 + (y + 12)^2 = 3m^2$

HOW THE DIAGRAM APPLIES

1. Center $(5, 12)$ goes in with flipped signs: $(x - 5)^2 + (y - 12)^2$. That rules out the two choices with $(x + 5)$ and $(y + 12)$.
2. The radius is $3m$, and the equation wants r^2 : $(3m)^2 = 9m^2$, not $3m^2$.
3. So the circle is $(x - 5)^2 + (y - 12)^2 = 9m^2$. Same idea in reverse: if a test gives you $= 49k^2$, the radius is $7k$, not $49k$.

ANSWER B $(x - 5)^2 + (y - 12)^2 = 9m^2$



Watch it solved

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